Solid State Fractional Capacitor:
A new circuit element

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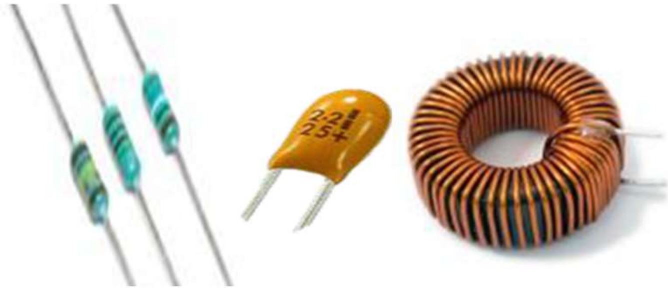
Circuit Element

$$Z(s) = \frac{Q}{s^{\alpha}}$$

$$Z(j\omega) = = \frac{Q}{\omega^{\alpha}} \angle\left(-\frac{\pi\alpha}{2}\right)$$



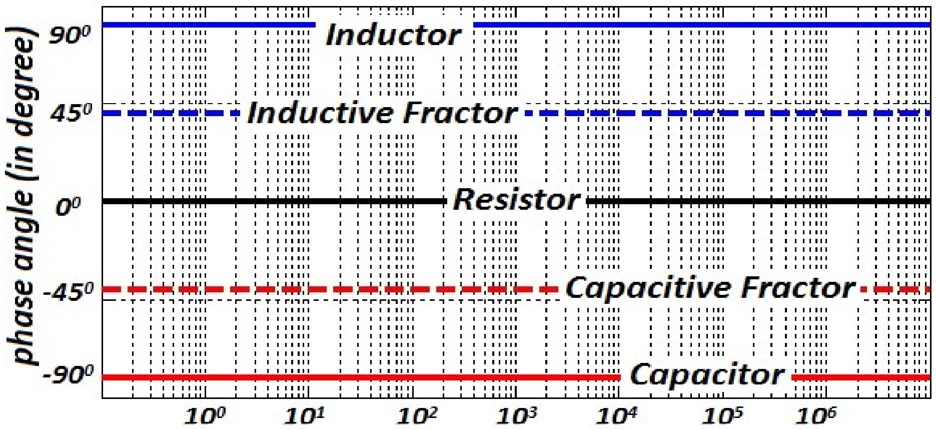
$\alpha = 0, \quad Z_R = Q \equiv R$

$\alpha = +1, \quad Z_C = \frac{Q}{s} \equiv \frac{1}{Cs}$

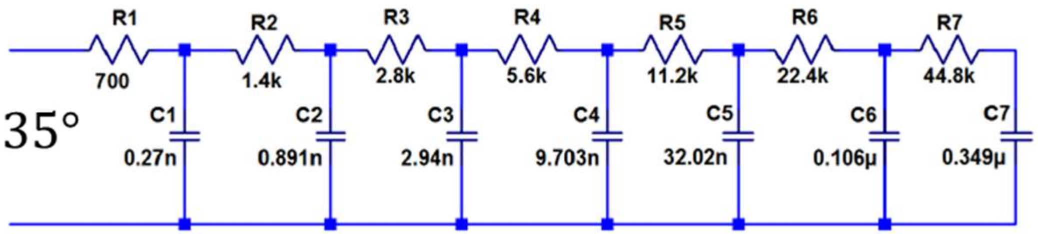
$\alpha = -1, \quad Z_L = Qs \equiv sL$

$\alpha : fraction, \quad Z_F = \frac{Q}{s^{\alpha}} \equiv \frac{1}{Fs^{\alpha}}$

$\alpha : fraction, \quad -90^0 < \phi = -\frac{\alpha\pi}{2} < +90^0$



$\varnothing = 35°$



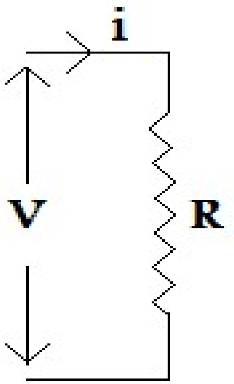
Svante Westerlund. Capacitor theory. *IEEE Transaction on Dielectrics and Electrical Insulation, 1(5): 826–839, October 1994*

Piotr Zoltowski. On the electrical capacitance of interfaces exhibiting constant phase element behavior. *Journal of Electroanalytical Chemistry 443, pages 149*–154, 1998

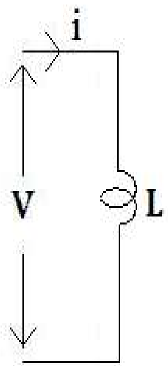
$$Z(s) = Qs^{-\alpha}$$

In Electrical Circuits

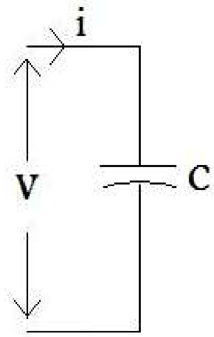
1. Resistance



2.Inductance



3. Capacitance

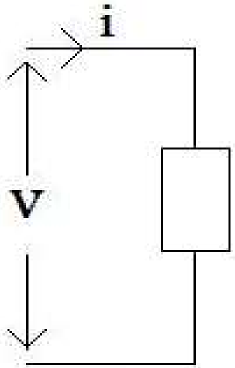


$$V = R\frac{d^0 i}{dt^0} = Ri \Rightarrow order \ '0'$$

$$V = L\frac{di}{dt} \Rightarrow order \ '1'$$

$$V = \frac{1}{C}\int_0^t idt \Rightarrow order \ '-1'$$

Fractional capacitance



$$V(s) = Qs^{-\alpha} I(s)$$

$-1<\alpha<1$

$$I(s) = \frac{1}{Q} s^{\alpha} V(s) + v(0)$$

$$V = Q\frac{d^{-\alpha} i}{dt^{-\alpha}},$$

Unifies the differentiation and integration: “Differintegral”

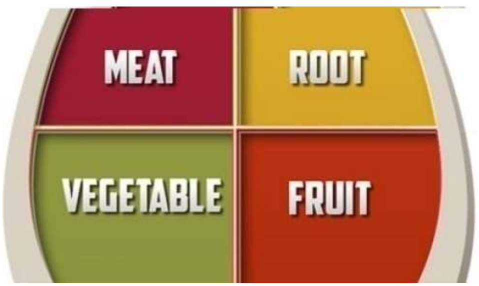
We can verify the 300 years old concept of fractional calculus mentioned by Leibnitz in 1695

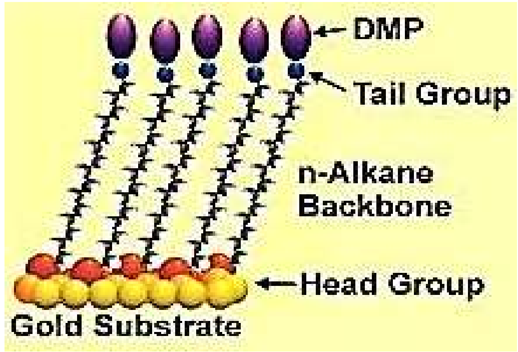
$$f(x) = x, \frac{d^n x}{dx^n}$$

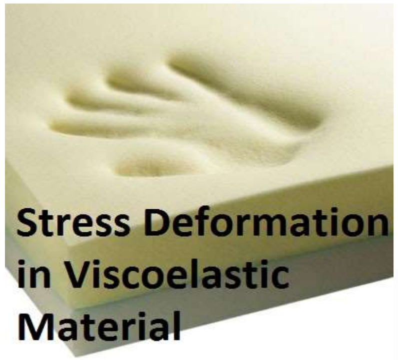
- **In a letter to L`Hospital in 1695 Leibniz raised the following question:** ***"Can the meaning of derivatives with integer order be generalized to derivatives with non-integer orders?"***

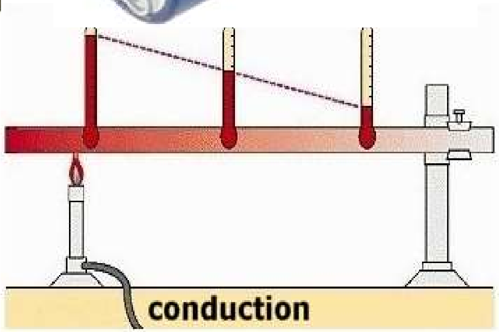
- **L`Hospital:*"What if the order will be 1/2?"***

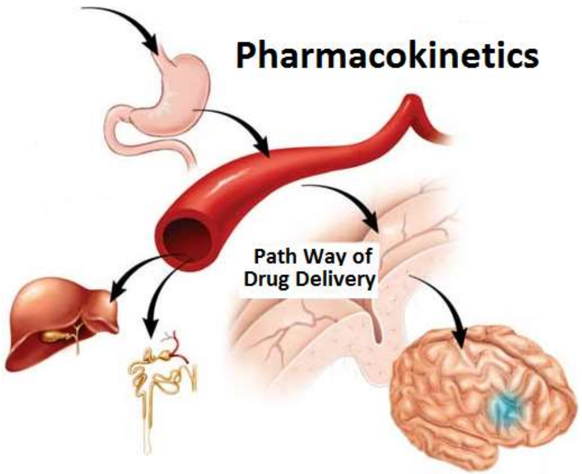
- **Leibniz replied:** ***"It will lead to a paradox, from which one day useful consequences will be drawn."***





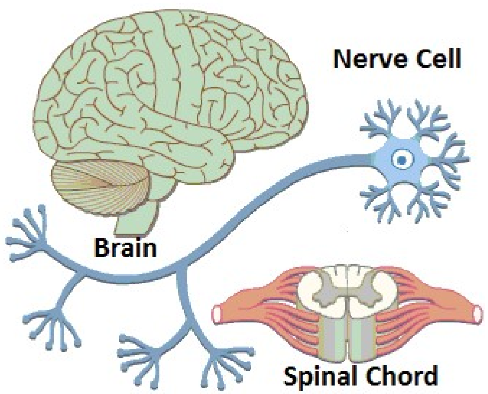


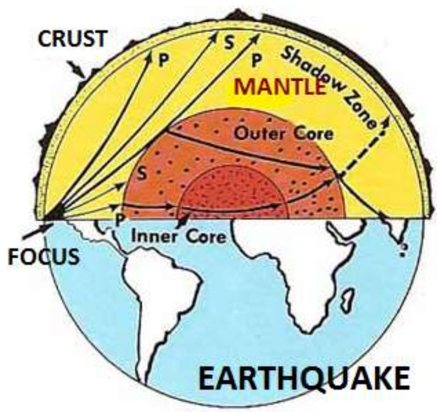




Nature is fractional not integer

Diffusion





In control Theory

Plant Structure:

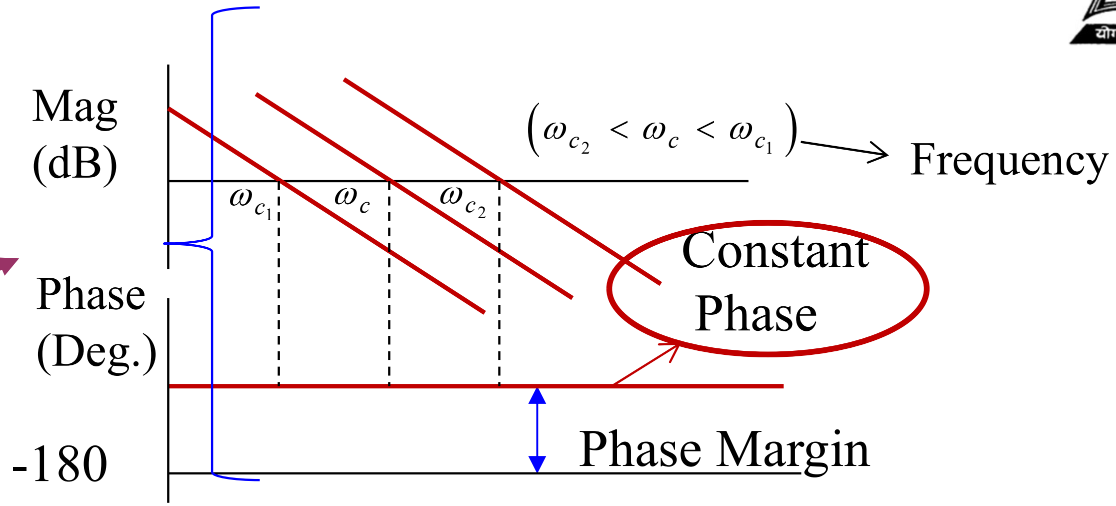
$$P(s) = \frac{K}{s^2 + a_1 s + a_0}$$

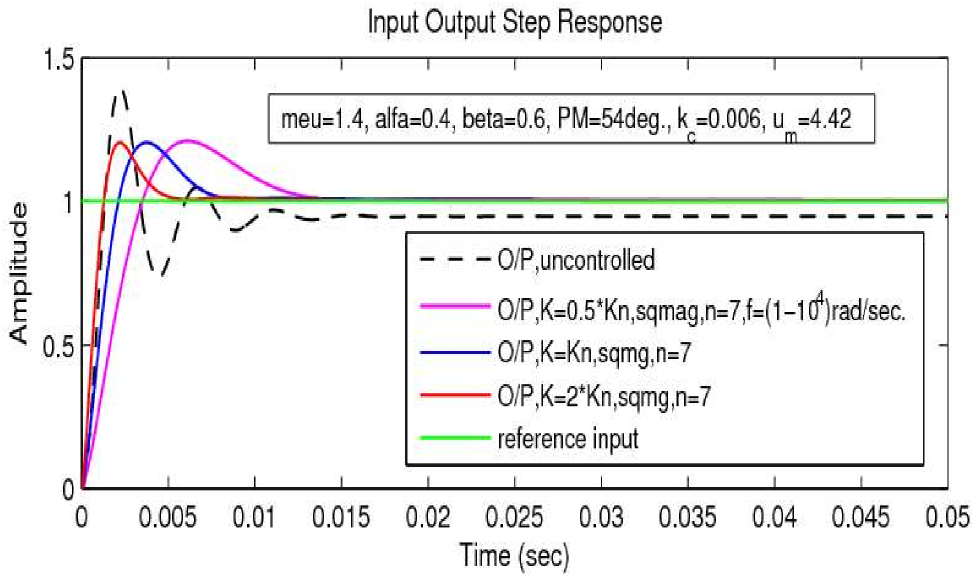
Desired Open Loop TF

$$P(s)C(s) = G(s) = \left(\frac{\omega_c}{s}\right)^{\gamma}$$

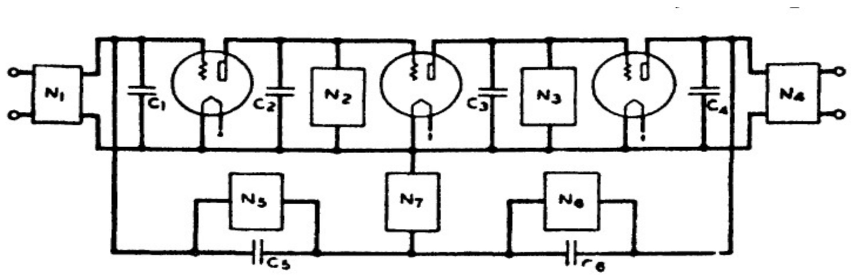
where $\gamma \in \Re^+$

Iso-damped Performance





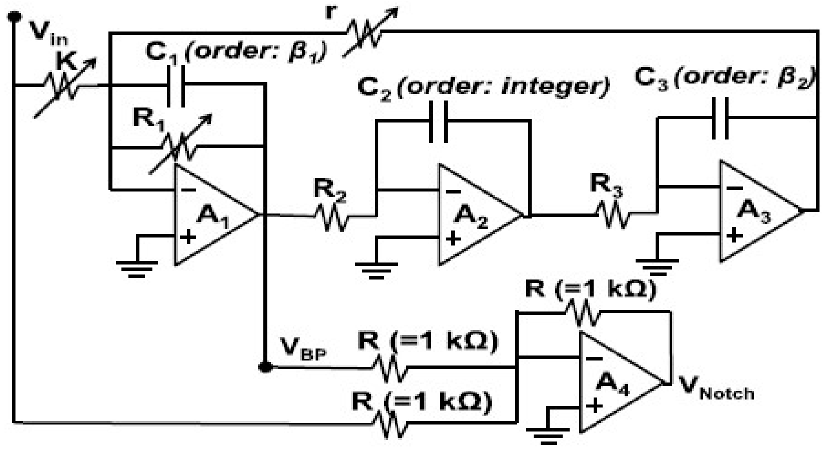
1945: Bode implemented a desired 60 degree phase control circuit using 19 elements (did not mention the term fractional)

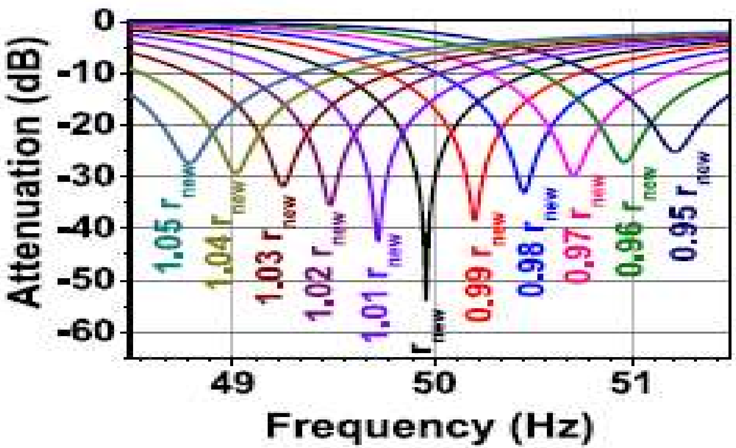


In last 80 years numerous technique was adopted to realize fractional order element using different RC ladder circuit

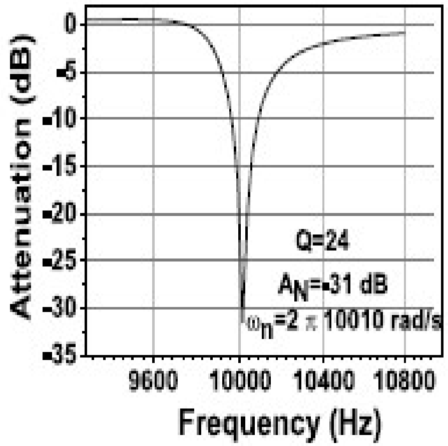
Notch filter for rejecting a single frequency

Fractional capacitor Name	Designed order (β)	β_{avg}	$\Delta\beta_{avg}$	Fractance ($\mho s^{\beta}$)
C_1	0.9	0.8948	0.03	896.16n
C_3	0.3	0.299	0.01	13.3μ

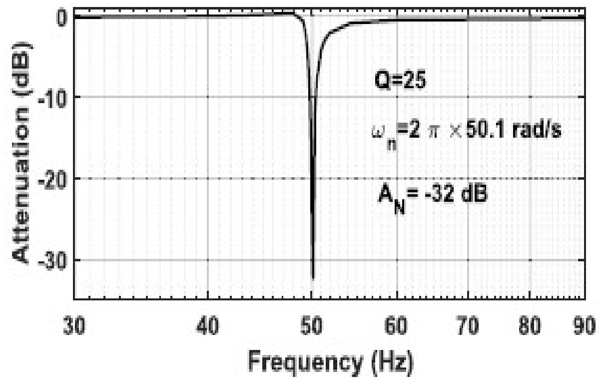




IONF/FONF	Different Notch Filters	A_N	Q
IONF	Current mode NF [29]	-55	10
FONF	Resonator based NF [9]	-35	13.5
IONF	Fliege Notch Filter [5]	-32	10
IONF	OTA based NF [30]	-26	1
IONF	Switched-capacitor NF [31]	-20	18.7
FONF	Proposed	-32	25



Experimental result



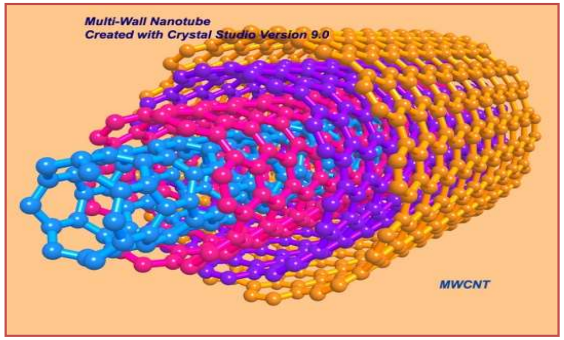
Arpit Sourav Mohapatra, and K Biswas, “A Fractional Order Notch Filter to Compensate the Attenuation-Loss Due to Change in Order of the Circuit” IEEE Transactions on Circuits and Systems I: Regular Papers 68 (2), 655-666, 2020.

We need solid state device to be used as a discrete circuit element

Fabrication

1. Materials

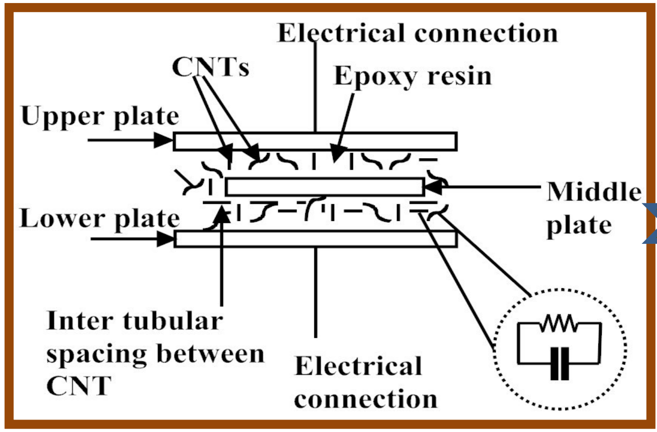
MWCNT



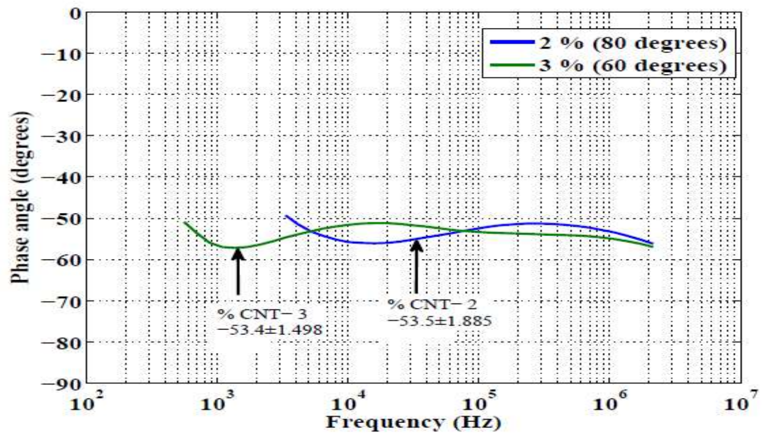
Epoxy resin and Hardener

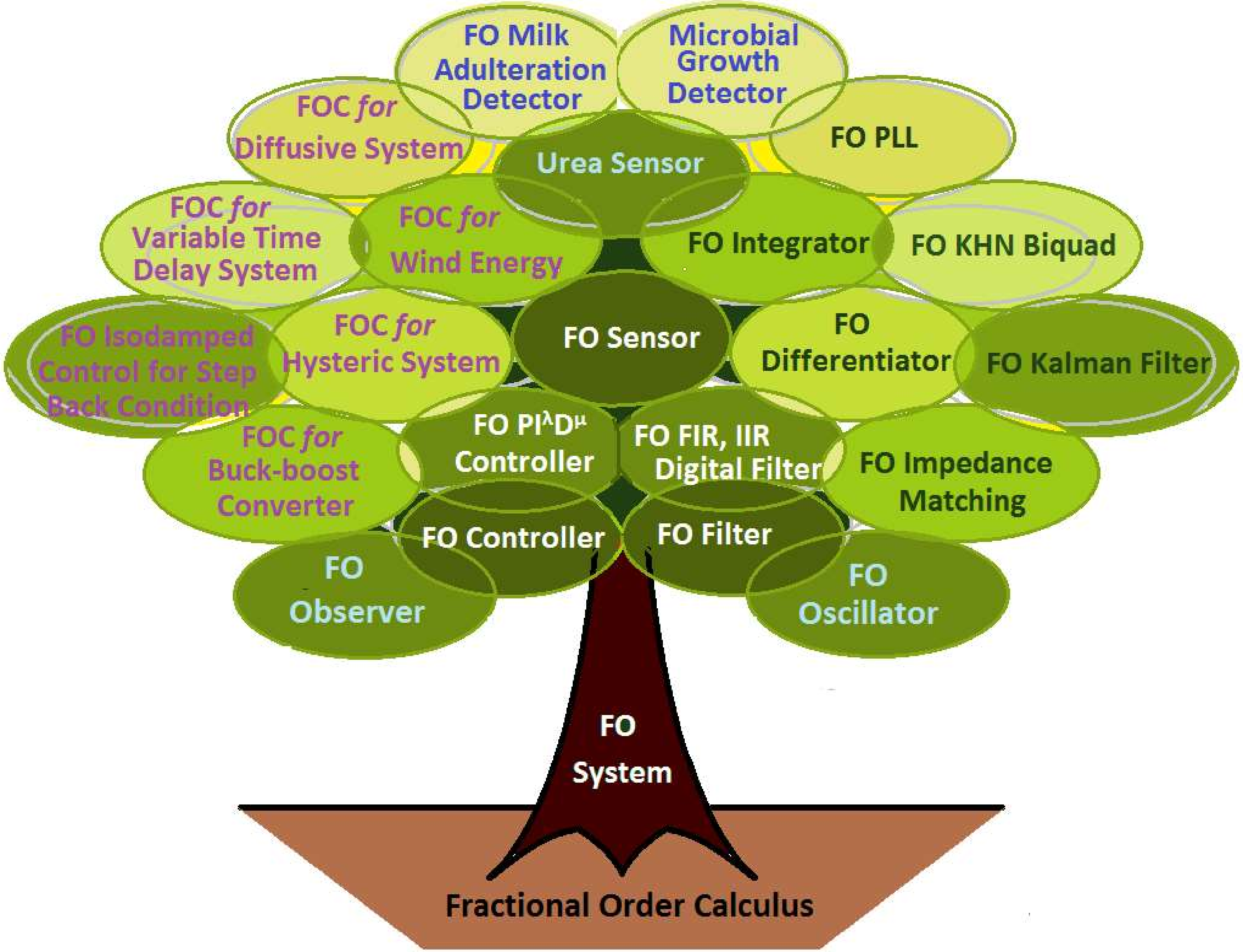
$$Z(s) = Qs^{-\alpha}$$

2. Electrode structure



3. Frequency response




FO Milk Adulteration Detector
Microbial Growth Detector
FOC *for* Diffusive System
FO PLL
Urea Sensor
FOC *for* Variable Time Delay System
FOC *for* Wind Energy
FO Integrator
FO KHN Biquad
FO Isodamped Control for Step Back Condition
FOC *for* Hysteric System
FO Sensor
FO Differentiator
FO Kalman Filter
FOC *for* Buck-boost Converter
FO PI$^{\lambda}$D$^{\mu}$ Controller
FO FIR, IIR Digital Filter
FO Impedance Matching
FO Controller
FO Filter
FO Observer
FO Oscillator
FO System
Fractional Order Calculus

Thank you